name	notation	SI	dimension
force	F	$\mathrm{N}=\frac{\mathrm{kg\cdot m}}{\mathrm{s^2}}$	$\mathrm{M^1L^1T^{-2}}$
energy	E,U	$\mathrm{J}=\mathrm{N\cdot m}$	$\mathrm{M^1L^2T^{-2}}$
power	P	$\mathrm{W}=\frac{\mathrm{J}}{\mathrm{s}}$	$\mathrm{M^1L^2T^{-3}}$
charge	q	$\mathrm{C}=\mathrm{A\cdot s}$	$\mathrm{T^1I^1}$
charge density	ρ,σ,λ	$\frac{\mathrm{C}}{\mathrm{m^{3,2,1}}}$	$\mathrm{L^{-3,-2,-1}T^1I^1}$
E field	$\boldsymbol{E}$	$\frac{\mathrm{N}}{\mathrm{C}}=\frac{\mathrm{V}}{\mathrm{m}}$	$\mathrm{M^1L^1T^{-3}I^{-1}}$
vacuum permittivity	ε_0	$\frac{\mathrm{F}}{\mathrm{m}}=\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$	$\mathrm{M^{-1}L^{-3}T^4I^2}$
E potential	φ,V	$\mathrm{V}=\frac{\mathrm{N\cdot m}}{\mathrm{C}}$	$\mathrm{M^1L^2T^{-3}I^{-1}}$
capacity	C	$\mathrm{F}=\frac{\mathrm{C}}{\mathrm{V}}$	$\mathrm{M^{-1}L^{-2}T^4I^2}$
E dipole moment	$\boldsymbol{p}$	$\mathrm{C\cdot m}$	$\mathrm{L^1T^1I^1}$
E quadrupole moment	$\boldsymbol{D}$	$\mathrm{C\cdot m^2}$	$\mathrm{L^2T^1I^1}$
polarization density	$\boldsymbol{P}$	$\frac{\mathrm{C}}{\mathrm{m^2}}$	$\mathrm{L^{-2}T^1I^1}$
E displacement field	$\boldsymbol{D}$	$\frac{\mathrm{C}}{\mathrm{m^2}}$	$\mathrm{L^{-2}T^1I^1}$
E susceptibility	χ	1	1
relative permittivity	κ,ε_r	1	1
resistivity	ρ	$\Omega\cdot\mathrm{m}$	$\mathrm{M^1L^3T^{-3}I^{-2}}$
conductivity	σ	$\frac{\mathrm{S}}{\mathrm{m}}$	$\mathrm{M^{-1}L^{-3}T^3I^2}$
current density	$\boldsymbol{j},\boldsymbol{J},\boldsymbol{I}$	$\frac{\mathrm{A}}{\mathrm{m^{2,1,0}}}$	$\mathrm{L^{-2,-1,0}I}$
M field	$\boldsymbol{B}$	$\mathrm{T}=\frac{\mathrm{N\cdot s}}{\mathrm{C\cdot m}}$	$\mathrm{M^1T^{-2}I^{-1}}$
M field	$\boldsymbol{H}$	$\frac{\mathrm{A}}{\mathrm{m}}$	$\mathrm{L^{-1}I}$
vacuum permeability	μ_0	$\frac{\mathrm{N}}{\mathrm{A^2}}$	$\mathrm{M^1L^1T^{-2}I^{-2}}$
M vector potential	$\boldsymbol{A}$	$\frac{\mathrm{V\cdot s}}{\mathrm{m}}$	$\mathrm{M^1L^1T^{-2}I^{-1}}$
M dipole moment	$\boldsymbol{\mu}$	$\mathrm{A\cdot m^2}$	$\mathrm{L^2I^1}$
M flux	Φ	$\mathrm{Wb}=\mathrm{T\cdot m^2}=\mathrm{V\cdot s}$	$\mathrm{M^1L^2T^{-2}I^{-2}}$
inductance	L,M	$\mathrm{H}=\frac{\mathrm{Wb}}{\mathrm{A}}$	$\mathrm{M^1L^2T^{-2}I^{-1}}$
impedance	Z,R,X	$\Omega=\frac{\mathrm{V}}{\mathrm{A}}$	$\mathrm{M^1L^2T^{-3}I^{-2}}$

$$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})=\boldsymbol{b}\cdot(\boldsymbol{c}\times\boldsymbol{a})=\boldsymbol{c}\cdot(\boldsymbol{a}\times\boldsymbol{b})$$

$$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$$

$$(\boldsymbol{a}\times\boldsymbol{b})\cdot(\boldsymbol{c}\times\boldsymbol{d})=(\boldsymbol{a}\cdot\boldsymbol{c})(\boldsymbol{b}\cdot\boldsymbol{d})-(\boldsymbol{a}\cdot\boldsymbol{d})(\boldsymbol{b}\cdot\boldsymbol{c})$$

$$\nabla\cdot(f\boldsymbol{A})=f\nabla\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\nabla f$$

$$\nabla\times(f\boldsymbol{A})=f\nabla\times\boldsymbol{A}+\nabla f\times\boldsymbol{A}$$

$$\nabla(\boldsymbol{A}\cdot\boldsymbol{B})=\boldsymbol{A}\times(\nabla\times\boldsymbol{B})+\boldsymbol{B}\times(\nabla\times\boldsymbol{A})+\boldsymbol{A}(\nabla\cdot\boldsymbol{B})+\boldsymbol{B}(\nabla\cdot\boldsymbol{A})$$

$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{B}\cdot(\nabla\times\boldsymbol{A})-\boldsymbol{A}\cdot(\nabla\times\boldsymbol{B})$$

$$\nabla\times(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{A}(\nabla\cdot\boldsymbol{B})-\boldsymbol{B}(\nabla\cdot\boldsymbol{A})+(\boldsymbol{B}\cdot\nabla)\boldsymbol{A}-(\boldsymbol{A}\cdot\nabla)\boldsymbol{B}$$

$$\nabla\times(\nabla f)=\boldsymbol{0}$$

$$\nabla\cdot(\nabla\times\boldsymbol{A})=0$$

$$\nabla\times(\nabla\times\boldsymbol{A})=\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A}$$

$$\oiint_{\partial V}\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{s}=\iiint_V(\nabla\cdot\boldsymbol{A})\mathrm{d}v$$

$$\oint_{\partial S}\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}=\iint_S(\nabla\times\boldsymbol{A})\cdot\mathrm{d}\boldsymbol{s}$$

$$\iiint\boldsymbol{A}\cdot\nabla f+\iiint f\nabla\cdot\boldsymbol{A}=\oiint f\boldsymbol{A}\mathrm{d}\boldsymbol{s}$$

$$\nabla r^n=nr^{n-2}\boldsymbol{r}$$

$$\nabla\cdot(r^n\boldsymbol{r})=(n+3)r^n$$

$$\nabla^2r^n=n(n+1)r^{n-2}$$

$$\nabla\cdot\frac{\boldsymbol{r}}{r^3}=-\nabla^2\frac{1}{r}=4\pi\delta^3(\boldsymbol{r})$$

$$\nabla\times(r^n\boldsymbol{r})=0$$

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases}x=r\sin\theta\cos\phi\\ y=r\sin\theta\sin\phi\\ z=r\cos\theta\end{cases}\qquad\begin{cases}\hat{\mathbf{x}}=\sin\theta\cos\phi\,\hat{\mathbf{r}}+\cos\theta\cos\phi\,\hat{\boldsymbol{\theta}}-\sin\phi\,\hat{\boldsymbol{\phi}}\\ \hat{\mathbf{y}}=\sin\theta\sin\phi\,\hat{\mathbf{r}}+\cos\theta\sin\phi\,\hat{\boldsymbol{\theta}}+\cos\phi\,\hat{\boldsymbol{\phi}}\\ \hat{\mathbf{z}}=\cos\theta\,\hat{\mathbf{r}}-\sin\theta\,\hat{\boldsymbol{\theta}}\end{cases}$$

$$\begin{cases}r=\sqrt{x^2+y^2+z^2}\\ \theta=\tan^{-1}\left(\sqrt{x^2+y^2}/z\right)\\ \phi=\tan^{-1}(y/x)\end{cases}\qquad\begin{cases}\hat{\mathbf{r}}=\sin\theta\cos\phi\,\hat{\mathbf{x}}+\sin\theta\sin\phi\,\hat{\mathbf{y}}+\cos\theta\,\hat{\mathbf{z}}\\ \hat{\boldsymbol{\theta}}=\cos\theta\cos\phi\,\hat{\mathbf{x}}+\cos\theta\sin\phi\,\hat{\mathbf{y}}-\sin\theta\,\hat{\mathbf{z}}\\ \hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}\end{cases}$$

Cylindrical

$$\begin{cases}x=s\cos\phi\\ y=s\sin\phi\\ z=z\end{cases}\qquad\begin{cases}\hat{\mathbf{x}}=\cos\phi\,\hat{\mathbf{s}}-\sin\phi\,\hat{\boldsymbol{\phi}}\\ \hat{\mathbf{y}}=\sin\phi\,\hat{\mathbf{s}}+\cos\phi\,\hat{\boldsymbol{\phi}}\\ \hat{\mathbf{z}}=\hat{\mathbf{z}}\end{cases}$$

$$\begin{cases}s=\sqrt{x^2+y^2}\\ \phi=\tan^{-1}(y/x)\\ z=z\end{cases}\qquad\begin{cases}\hat{\mathbf{s}}=\cos\phi\,\hat{\mathbf{x}}+\sin\phi\,\hat{\mathbf{y}}\\ \hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}\\ \hat{\mathbf{z}}=\hat{\mathbf{z}}\end{cases}$$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l}=dx\,\hat{\mathbf{x}}+dy\,\hat{\mathbf{y}}+dz\,\hat{\mathbf{z}};\quad d\tau=dx\,dy\,dz$

Gradient: $\nabla t=\frac{\partial t}{\partial x}\hat{\mathbf{x}}+\frac{\partial t}{\partial y}\hat{\mathbf{y}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla\cdot\mathbf{v}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}$

Curl: $\nabla\times\mathbf{v}=\left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}}+\left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}}+\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\nabla^2t=\frac{\partial^2t}{\partial x^2}+\frac{\partial^2t}{\partial y^2}+\frac{\partial^2t}{\partial z^2}$

Spherical. $d\mathbf{l}=dr\,\hat{\mathbf{r}}+r\,d\theta\,\hat{\boldsymbol{\theta}}+r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}};\quad d\tau=r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\nabla t=\frac{\partial t}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\boldsymbol{\theta}}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}}$

Divergence: $\nabla\cdot\mathbf{v}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,v_\theta)+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

Curl: $\nabla\times\mathbf{v}=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,v_\phi)-\frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}}$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{\partial}{\partial r}(rv_\phi)\right]\hat{\boldsymbol{\theta}}+\frac{1}{r}\left[\frac{\partial}{\partial r}(rv_\theta)-\frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $\nabla^2t=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2t}{\partial\phi^2}$

Cylindrical. $d\mathbf{l}=ds\,\hat{\mathbf{s}}+s\,d\phi\,\hat{\boldsymbol{\phi}}+dz\,\hat{\mathbf{z}};\quad d\tau=s\,ds\,d\phi\,dz$

Gradient: $\nabla t=\frac{\partial t}{\partial s}\hat{\mathbf{s}}+\frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla\cdot\mathbf{v}=\frac{1}{s}\frac{\partial}{\partial s}(sv_s)+\frac{1}{s}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$

Curl: $\nabla\times\mathbf{v}=\left[\frac{1}{s}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}}+\left[\frac{\partial v_s}{\partial z}-\frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}}+\frac{1}{s}\left[\frac{\partial}{\partial s}(sv_\phi)-\frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$

Laplacian: $\nabla^2t=\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right)+\frac{1}{s^2}\frac{\partial^2t}{\partial\phi^2}+\frac{\partial^2t}{\partial z^2}$

Proof. Frame S: the wire is at rest, the test charge $-q$ with velocity v_0 is at distance r from the axis, positive charges are at rest, negative charges move with velocity v. Let the line charge density be $\lambda_+=\lambda_-=\lambda$, the force be F.

Frame S': the test charge is at rest. Let the line charge density be λ'_+ and λ'_-, the force be F'.

Let the ‘inherent line charge density’ be λ_{0+} and λ_{0-}. Let

$$\gamma(v)=\frac{1}{\sqrt{1-v^2/c^2}}.$$

Transformation of line charge density: $l=l_0/\gamma(v)\Longrightarrow\lambda=\gamma(v)\lambda_0$.

We have:

$$\begin{cases}\lambda=\lambda_{0+}\\ \lambda=\gamma(v)\lambda_{0-}\\ F=qv_0B=qv_0\frac{\mu_0I}{2\pi r}=\frac{q\mu_0\lambda vv_0}{2\pi r}\\ \lambda'_+=\gamma(-v_0)\lambda_{0+}\\ \lambda'_-=\gamma\left(\frac{v-v_0}{1-vv_0/c^2}\right)\lambda_{0-}\\ F'=\frac{q}{2\pi\varepsilon_0r}\left(\lambda'_+-\lambda'_-\right)\end{cases}$$

Solve it, we get

$$F'=\gamma(v_0)\frac{q\lambda vv_0}{2\pi c^2\varepsilon_0r},$$

which is consistent with the transformation of force.

$$\begin{cases}Z_{Y1}=\frac{Z_{\Delta2}Z_{\Delta3}}{Z_{\Delta1}+Z_{\Delta2}+Z_{\Delta3}}\\ Z_{Y2}=\frac{Z_{\Delta1}Z_{\Delta3}}{Z_{\Delta1}+Z_{\Delta2}+Z_{\Delta3}}\\ Z_{Y3}=\frac{Z_{\Delta1}Z_{\Delta2}}{Z_{\Delta1}+Z_{\Delta2}+Z_{\Delta3}}\end{cases}$$

$$\begin{cases}Z_{\Delta1}=Z_{Y2}+Z_{Y3}+\frac{Z_{Y2}Z_{Y3}}{Z_{Y1}}\\ Z_{\Delta2}=Z_{Y1}+Z_{Y3}+\frac{Z_{Y1}Z_{Y3}}{Z_{Y2}}\\ Z_{\Delta3}=Z_{Y1}+Z_{Y2}+\frac{Z_{Y1}Z_{Y2}}{Z_{Y3}}\end{cases}$$

$$-\frac{\sigma_0}{3\varepsilon_0}\hat{\boldsymbol{x}};\ \frac{R^3\sigma_0}{3\varepsilon_0}\frac{3\boldsymbol{r}_x\boldsymbol{r}-r^2\hat{\boldsymbol{x}}}{r^5}=\frac{R^3\sigma_0}{\varepsilon_0\boldsymbol{r}^3}\left(\cos\theta-\frac{1}{3},\cos\theta\sin\theta\right)$$

$$\begin{cases}E=E_0-\frac{P}{3\varepsilon_0}\\ P=\varepsilon_0(\varepsilon_r-1)E\end{cases}$$

$$\begin{cases}E=\frac{3\varepsilon_0}{\varepsilon_r+2}E_0\\ P=\frac{\varepsilon_r-1}{\varepsilon_r+2}3\varepsilon_0E_0\end{cases}$$

$$\boldsymbol{E}_1=\boldsymbol{E}_2=\frac{Q\boldsymbol{r}}{2\pi(\varepsilon_1+\varepsilon_2)r^3}$$

$$\boldsymbol{D}_1=\frac{\varepsilon_1Q\boldsymbol{r}}{2\pi(\varepsilon_1+\varepsilon_2)r^3},\qquad\boldsymbol{D}_1=\frac{\varepsilon_2Q\boldsymbol{r}}{2\pi(\varepsilon_1+\varepsilon_2)r^3}$$

看作由两个半球形电容器并联而成，所以总

$$C=\frac{2\pi(\varepsilon_1+\varepsilon_2)ab}{b-a}$$

Maxwell : $\nabla\cdot\boldsymbol{E}=\dfrac{\rho}{\varepsilon_0}$ $\nabla\times\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t}$ $\nabla\cdot\boldsymbol{B}=0$ $\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}+\varepsilon_0\mu_0\dfrac{\partial\boldsymbol{E}}{\partial t}$ $\varepsilon_0\mu_0=\dfrac{1}{c^2}$ $\nabla\cdot\boldsymbol{D}=\rho_{\text{free}}$ $\nabla\times\boldsymbol{H}=\boldsymbol{j}_{\text{free}}+\dfrac{\partial\boldsymbol{D}}{\partial t}$

$\varepsilon_0\mu_0c^2=1$ Lorentz force : $\boldsymbol{F}=q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$ $\dfrac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}l}=\boldsymbol{I}\times\boldsymbol{B}$

electric : $\boldsymbol{E}=\iiint\dfrac{\rho(\boldsymbol{r'})\mathrm{d}v}{4\pi\varepsilon_0}\cdot\dfrac{\boldsymbol{r}-\boldsymbol{r'}}{|\boldsymbol{r}-\boldsymbol{r'}|^3}$ $\varphi=\iiint\dfrac{\rho(\boldsymbol{r'})\mathrm{d}v}{4\pi\varepsilon_0|\boldsymbol{r}-\boldsymbol{r'}|}$ $\oiint\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{s}=\dfrac{Q}{\varepsilon_0}$ $C=\dfrac{Q}{V}$ $\varphi=\dfrac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\varepsilon_0r^3}$ $\boldsymbol{E}=\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{3(\boldsymbol{p}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5}-\dfrac{\boldsymbol{p}}{r^3}\right)$

$\boldsymbol{P}=Nq\boldsymbol{\delta}=\chi\varepsilon_0\boldsymbol{E}$ $\sigma_{\text{pol}}=\boldsymbol{P}\cdot\boldsymbol{n}$ $\rho_{\text{pol}}=-\nabla\cdot\boldsymbol{P}$ $\boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P}=\kappa\varepsilon_0\boldsymbol{E}$ $\boldsymbol{P}=\left(1-\dfrac{1}{\kappa}\right)\boldsymbol{D}=(\kappa-1)\varepsilon_0\boldsymbol{E}$

VP : $\boldsymbol{B}=\nabla\times\boldsymbol{A}$ $\nabla\cdot\boldsymbol{A}=0$ $\nabla^2\boldsymbol{A}=-\mu_0\boldsymbol{j}$ $\boldsymbol{A}=\dfrac{\mu_0}{4\pi}\iiint\dfrac{\boldsymbol{j}\mathrm{d}v}{|\boldsymbol{r}-\boldsymbol{r'}|}$ $\boldsymbol{B}=\dfrac{\mu_0}{4\pi}\iiint\dfrac{\boldsymbol{j}\mathrm{d}v\times(\boldsymbol{r}-\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|^3}$ $\oint\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l}=\mu_0\iint\boldsymbol{j}\cdot\mathrm{d}\boldsymbol{s}=\mu_0\boldsymbol{I}$

current : $\boldsymbol{j}=\rho\boldsymbol{v}$ $\boldsymbol{j}\mathrm{d}v=\boldsymbol{J}\mathrm{d}s=I\mathrm{d}\boldsymbol{l}$ $\nabla\cdot\boldsymbol{j}=-\dfrac{\partial\rho}{\partial t}$ ring : $\boldsymbol{B}=\dfrac{\mu_0Ir^2}{2\left(r^2+z^2\right)^{3/2}}$ resistance : $R=\rho\dfrac{l}{S}$ $\boldsymbol{j}=\sigma\boldsymbol{E}$

wire : $\boldsymbol{B}=\dfrac{\mu_0I}{2\pi r}\hat{\boldsymbol{\phi}}$ $\boldsymbol{A}=-\dfrac{\mu_0I\ln r}{2\pi}\hat{\boldsymbol{z}}$ solenoid : $\boldsymbol{B}=\begin{cases}\mu_0J\hat{\boldsymbol{z}}, & r<R\\ \boldsymbol{0}, & r>R\end{cases}$ $\boldsymbol{A}=\begin{cases}\dfrac{\mu_0Jr}{2}\hat{\boldsymbol{\phi}}, & r<R\\ \dfrac{\mu_0JR^2}{2r}\hat{\boldsymbol{\phi}}, & r>R\end{cases}$ $L=\mu_0n^2V$

dipole : $\boldsymbol{\mu}=I\boldsymbol{S}$ $\boldsymbol{A}=\dfrac{\mu_0}{4\pi}\dfrac{\boldsymbol{\mu}\times\hat{\boldsymbol{r}}}{r^2}$ $\boldsymbol{B}=\dfrac{\mu_0}{4\pi}\left(\dfrac{3(\boldsymbol{\mu}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5}-\dfrac{\boldsymbol{\mu}}{r^3}\right)$ $\boldsymbol{\mu}=\dfrac{1}{2}\iiint\boldsymbol{r}\times\boldsymbol{j}\mathrm{d}v$ $W=\pm\boldsymbol{\mu}\cdot\boldsymbol{B}$

energy : $U=\dfrac{1}{2}CV^2$ $U=\dfrac{1}{2}\iiint\varphi\rho\mathrm{d}v=\dfrac{\varepsilon_0}{2}\iiint E^2\mathrm{d}v$ $U=\dfrac{1}{2}\iiint\boldsymbol{A}\cdot\boldsymbol{j}\mathrm{d}v=\dfrac{1}{2\mu_0}\iiint\boldsymbol{B}^2\mathrm{d}v=\dfrac{1}{2}\sum M_{i,j}I_iI_j$

inductance : $\Phi=\iint\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s}$ $\mathcal{E}=-\dfrac{\mathrm{d}\Phi}{\mathrm{d}t}=-L\dfrac{\mathrm{d}I}{\mathrm{d}t}-M\dfrac{\mathrm{d}I'}{\mathrm{d}t}$ $M=\dfrac{\mu_0}{4\pi}\oint\oint\dfrac{\mathrm{d}\boldsymbol{l}_1\cdot\mathrm{d}\boldsymbol{l}_2}{r_{12}}$ $M=k\sqrt{L_1L_2}$

displacement : $\boldsymbol{j}_d=\varepsilon_0\dfrac{\partial\boldsymbol{E}}{\partial t}$ new potential : $\boldsymbol{E}=-\nabla\phi-\dfrac{\partial\boldsymbol{A}}{\partial t}$ $\boldsymbol{B}=\nabla\times\boldsymbol{A}$ $\nabla\cdot\boldsymbol{A}+\mu_0\varepsilon_0\dfrac{\partial\phi}{\partial t}=0$

new equation : $\Box^2=\nabla^2-\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}$ $\Box^2\varphi=-\dfrac{\rho}{\varepsilon_0}$ $\Box^2\boldsymbol{A}=-\mu_0\boldsymbol{j}$ $\Box^2\boldsymbol{E}=\dfrac{\nabla\rho}{\varepsilon_0}+\mu_0\dfrac{\partial\boldsymbol{j}}{\partial t}$ $\Box^2\boldsymbol{B}=-\mu_0\nabla\times\boldsymbol{j}$

planar wave : $\dfrac{\partial^2\psi}{\partial x^2}-\dfrac{1}{c^2}\dfrac{\partial^2\psi}{\partial t^2}=0$ $\boldsymbol{E}_y=f_1(x-ct)+g_1(x+ct)$ $c\boldsymbol{B}_z=f_1(x-ct)-g_1(x+ct)$ $c\boldsymbol{B}_y=-f_2(x-ct)+g_2(x+ct)$

spherical wave : $\dfrac{\partial^2\psi}{\partial r^2}+\dfrac{2}{r}\dfrac{\partial\psi}{\partial r}-\dfrac{1}{c^2}\dfrac{\partial^2\psi}{\partial t^2}=0$ $\psi=\dfrac{f(r-ct)}{r}\hat{\boldsymbol{r}}$ Helmholtz : $(\nabla^2+k^2)\boldsymbol{F}=0$

AC : $P=\dfrac{1}{2}I_0^2R$ $z_0=\dfrac{z_1}{2}\pm\sqrt{\dfrac{z_1^2}{4}+z_1z_2}$ low pass LC : $z_0=\dfrac{\mathrm{i}\omega L}{2}+\sqrt{\dfrac{L}{C}-\dfrac{\omega^2L^2}{4}}$ high pass CL : $z_0=\dfrac{1}{2\mathrm{i}\omega C}+\sqrt{\dfrac{L}{C}-\dfrac{1}{4\omega^2C^2}}$

boundary : $\boldsymbol{n}\cdot(\boldsymbol{E}_1-\boldsymbol{E}_2)=\dfrac{\sigma}{\varepsilon_0}$ $\boldsymbol{n}\times(\boldsymbol{E}_1-\boldsymbol{E}_2)=\boldsymbol{0}$ $\boldsymbol{n}\cdot(\boldsymbol{B}_1-\boldsymbol{B}_2)=\boldsymbol{0}$ $\boldsymbol{n}\times(\boldsymbol{B}_1-\boldsymbol{B}_2)=\mu_0\boldsymbol{J}$

rect res : $\boldsymbol{E}_x=A_x\mathrm{e}^{\mathrm{i}\omega t}\cos k_xx\sin k_yy\sin k_zz$ $\boldsymbol{E}_y=A_y\mathrm{e}^{\mathrm{i}\omega t}\sin k_xx\cos k_yy\sin k_zz$ $\boldsymbol{E}_z=A_z\mathrm{e}^{\mathrm{i}\omega t}\sin k_xx\sin k_yy\cos k_zz$

$-\mathrm{i}\omega\boldsymbol{B}=\nabla\times\boldsymbol{E}$ $\dfrac{\omega^2}{c^2}=k^2=k_x^2+k_y^2+k_z^2$ $0=A_xk_x+A_yk_y+A_zk_z$ min $\mathrm{TE}_{101},\mathrm{TM}_{110}$, no $\mathrm{TE}_{\mathrm{mn0}},\mathrm{TM}_{\mathrm{0np}},\mathrm{TM}_{\mathrm{m0p}}$

$f=\dfrac{\omega}{2\pi}$ $T=\dfrac{2\pi}{\omega}$ $k_c^2=k^2-\beta^2$ $\omega=c\pi\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{d^2}}$ waveguide : $\boldsymbol{E}=(\bar{\boldsymbol{e}}(x,y)+\hat{\boldsymbol{z}}e_z(x,y))\mathrm{e}^{\mathrm{i}(\omega t-\beta z)}$

$\boldsymbol{E}_x=\dfrac{-\mathrm{i}}{k_c^2}\left(\beta\dfrac{\partial\boldsymbol{E}_z}{\partial x}+\omega\dfrac{\partial\boldsymbol{B}_z}{\partial y}\right)$ $\boldsymbol{E}_y=\dfrac{\mathrm{i}}{k_c^2}\left(-\beta\dfrac{\partial\boldsymbol{E}_z}{\partial y}+\omega\dfrac{\partial\boldsymbol{B}_z}{\partial x}\right)$ $\boldsymbol{B}_x=\dfrac{\mathrm{i}}{k_c^2}\left(\varepsilon_0\mu_0\omega\dfrac{\partial\boldsymbol{E}_z}{\partial y}-\beta\dfrac{\partial\boldsymbol{B}_z}{\partial x}\right)$ $\boldsymbol{B}_y=\dfrac{-\mathrm{i}}{k_c^2}\left(\varepsilon_0\mu_0\omega\dfrac{\partial\boldsymbol{E}_z}{\partial x}+\beta\dfrac{\partial\boldsymbol{B}_z}{\partial y}\right)$

$Z=\dfrac{\boldsymbol{E}_x}{\boldsymbol{H}_y}=-\dfrac{\boldsymbol{E}_y}{\boldsymbol{H}_x}$ $\boldsymbol{H}=\dfrac{\boldsymbol{B}}{\mu_0}$ $Z_{\text{TEM}}=\dfrac{\mu_0\omega}{\beta}=\dfrac{\mu_0\omega}{k}=\mu_0c=\sqrt{\dfrac{\mu_0}{\varepsilon_0}}=\eta_0$ $Z_{\text{TE}}=\dfrac{\mu_0\omega}{\beta}=\dfrac{k\eta_0}{\beta}$ $Z_{\text{TM}}=\dfrac{\beta}{\varepsilon_0\omega}=\dfrac{\beta\eta_0}{k}$ $\lambda_g=\dfrac{2\pi}{\beta}$

$v_p=\dfrac{\partial z}{\partial t}=\dfrac{\omega}{\beta}=\dfrac{k}{\beta}c$ $v_g=\dfrac{\partial\omega}{\partial\beta}=\dfrac{\beta c^2}{\omega}=\dfrac{\beta}{k}c$ coaxial wg : $\gamma=\sqrt{(R+\mathrm{i}\omega L)(G+\mathrm{i}\omega C)}$ $R=G=0\Rightarrow\beta=\omega\sqrt{LC}$ $Z=\sqrt{L/C}$

rect wg : $\beta=\sqrt{k^2-\left(\dfrac{m\pi}{a}\right)^2-\left(\dfrac{n\pi}{b}\right)^2}$ $f_{c_{mn}}=\dfrac{\omega_c}{2\pi}=\dfrac{ck}{2\pi}=\dfrac{ck_c}{2\pi}=\dfrac{1}{2\pi\sqrt{\varepsilon_0\mu_0}}\sqrt{\left(\dfrac{m\pi}{a}\right)^2+\left(\dfrac{n\pi}{b}\right)^2}$

TE_{mn} : $\boldsymbol{E}_x=\dfrac{\mathrm{i}\omega n\pi}{k_c^2b}A\cos\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{E}_y=\dfrac{-\mathrm{i}\omega m\pi}{k_c^2a}A\sin\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{E}_z=0$

$\boldsymbol{B}_x=\dfrac{\mathrm{i}\beta m\pi}{k_c^2a}A\sin\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{B}_y=\dfrac{\mathrm{i}\beta n\pi}{k_c^2b}A\cos\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{B}_z=A\cos\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$

TM_{mn} : $\boldsymbol{E}_x=\dfrac{-\mathrm{i}\beta m\pi}{k_c^2a}A\cos\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{E}_y=\dfrac{-\mathrm{i}\beta n\pi}{k_c^2b}A\sin\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{E}_z=A\sin\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$

$\boldsymbol{B}_x=\dfrac{\mathrm{i}\varepsilon_0\mu_0\omega n\pi}{k_c^2b}A\sin\dfrac{m\pi x}{a}\cos\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{B}_y=\dfrac{-\mathrm{i}\varepsilon_0\mu_0\omega m\pi}{k_c^2a}A\cos\dfrac{m\pi x}{a}\sin\dfrac{n\pi y}{b}\mathrm{e}^{-\mathrm{i}\beta z}$ $\boldsymbol{B}_z=0$

transform : $x'=\gamma(x-vt)$ $t'=\gamma(t-vx/c^2)$ $\boldsymbol{j}'_x=\gamma(\boldsymbol{j}_x-v\rho)$ $\rho'=\gamma(\rho-v\boldsymbol{j}_x/c^2)$ $F'=\gamma F$ $v'=\dfrac{v+v_0}{\frac{v_0v}{c^2}+1}$

General Physics II Final Cheatsheet