General Physics II Midterm Cheatsheet

$$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})=\boldsymbol{b}\cdot(\boldsymbol{c}\times\boldsymbol{a})=\boldsymbol{c}\cdot(\boldsymbol{a}\times\boldsymbol{b})$$

$$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$$

$$(\boldsymbol{a}\times\boldsymbol{b})\cdot(\boldsymbol{c}\times\boldsymbol{d})=(\boldsymbol{a}\cdot\boldsymbol{c})(\boldsymbol{b}\cdot\boldsymbol{d})-(\boldsymbol{a}\cdot\boldsymbol{d})(\boldsymbol{b}\cdot\boldsymbol{c})$$

$$\nabla\cdot(f\boldsymbol{A})=f\nabla\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\nabla f$$

$$\nabla\times(f\boldsymbol{A})=f\nabla\times\boldsymbol{A}+\nabla f\times\boldsymbol{A}$$

$$\nabla(\boldsymbol{A}\cdot\boldsymbol{B})=\boldsymbol{A}\times(\nabla\times\boldsymbol{B})+\boldsymbol{B}\times(\nabla\times\boldsymbol{A})+\boldsymbol{A}(\nabla\cdot\boldsymbol{B})+\boldsymbol{B}(\nabla\cdot\boldsymbol{A})$$

$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{B}\cdot(\nabla\times\boldsymbol{A})-\boldsymbol{A}\cdot(\nabla\times\boldsymbol{B})$$

$$\nabla\times(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{A}(\nabla\cdot\boldsymbol{B})-\boldsymbol{B}(\nabla\cdot\boldsymbol{A})+(\boldsymbol{B}\cdot\nabla)\boldsymbol{A}-(\boldsymbol{A}\cdot\nabla)\boldsymbol{B}$$

$$\nabla\times(\nabla f)=\boldsymbol{0}$$

$$\nabla\cdot(\nabla\times\boldsymbol{A})=0$$

$$\nabla\times(\nabla\times\boldsymbol{A})=\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A}$$

$$\oiint_{\partial V}\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{s}=\iiint_V(\nabla\cdot\boldsymbol{A})\mathrm{d}v$$

$$\oint_{\partial S}\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}=\iint_S(\nabla\times\boldsymbol{A})\cdot\mathrm{d}\boldsymbol{s}$$

$$\nabla r^n=nr^{n-2}\boldsymbol{r}$$

$$\nabla\cdot(r^n\boldsymbol{r})=(n+3)r^n$$

$$\nabla^2r^n=n(n+1)r^{n-2}$$

$$\nabla\cdot\frac{\boldsymbol{r}}{r^3}=-\nabla^2\frac{1}{r}=4\pi\delta^3(\boldsymbol{r})$$

$$\nabla\times(r^n\boldsymbol{r})=0$$

$$\mathrm{Exp}(x+\mathrm{i}y)=\mathrm{e}^x(\cos y+\mathrm{i}\sin y)$$

$$\mathrm{Ln}\,z=\ln r+\mathrm{i}\theta=\ln|z|+\mathrm{i}\,\mathrm{Arg}\,z=\ln\sqrt{x^2+y^2}+\mathrm{i}\,\mathrm{atan2}(y,x)$$

analytic function :

$$\begin{cases}\dfrac{\partial U}{\partial x}=\dfrac{\partial V}{\partial y}\\[2mm]\dfrac{\partial U}{\partial y}=-\dfrac{\partial V}{\partial x}\end{cases}\Longrightarrow\begin{cases}\nabla^2U=0\\\nabla^2V=0\end{cases}\Longrightarrow\nabla U\cdot\nabla V=0$$

$$\varphi(z)=\phi(w(z))$$

$$\mathrm{N}=\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2},\ \mathrm{C}=\mathrm{A}\cdot\mathrm{s},\ \mathrm{F}=\frac{\mathrm{C}}{\mathrm{V}},\ \mathrm{J}=\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}$$

name	notation	SI	dimension
charge	q	C	$\mathrm{T}^1\mathrm{I}^1$
charge density	ρ,σ,λ	$\frac{\mathrm{C}}{\mathrm{m}^{3,2,1}}$	$\mathrm{L}^{-3,-2,-1}\mathrm{T}^1\mathrm{I}^1$
E field	E	$\frac{\mathrm{N}}{\mathrm{C}}=\frac{\mathrm{V}}{\mathrm{m}}$	$\mathrm{M}^1\mathrm{L}^1\mathrm{T}^{-3}\mathrm{I}^{-1}$
vacuum permittivity	ε_0	$\frac{\mathrm{F}}{\mathrm{m}}=\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$	$\mathrm{M}^{-1}\mathrm{L}^{-3}\mathrm{T}^4\mathrm{I}^2$
E potential	φ,V	$\mathrm{V}=\frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{C}}$	$\mathrm{M}^1\mathrm{L}^2\mathrm{T}^{-3}\mathrm{I}^{-1}$
capacity	C	$\mathrm{F}=\frac{\mathrm{C}}{\mathrm{V}}$	$\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{T}^4\mathrm{I}^2$
dipole moment	p	$\mathrm{C}\cdot\mathrm{m}$	$\mathrm{L}^1\mathrm{T}^1\mathrm{I}^1$
quadrupole moment	d	$\mathrm{C}\cdot\mathrm{m}^2$	$\mathrm{L}^2\mathrm{T}^1\mathrm{I}^1$
electrostatic energy	U	$\mathrm{J}=\mathrm{N}\cdot\mathrm{m}$	$\mathrm{M}^1\mathrm{L}^2\mathrm{T}^{-2}$
polarization density	P	$\frac{\mathrm{C}}{\mathrm{m}^2}$	$\mathrm{L}^{-2}\mathrm{T}^1\mathrm{I}^1$
E displacement field	D	$\frac{\mathrm{C}}{\mathrm{m}^2}$	$\mathrm{L}^{-2}\mathrm{T}^1\mathrm{I}^1$
E susceptibility	χ	1	1
relative permittivity	κ,ε_r	1	1

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases}x=r\sin\theta\cos\phi\\y=r\sin\theta\sin\phi\\z=r\cos\theta\end{cases}\qquad\begin{cases}\hat{\mathbf{x}}=\sin\theta\cos\phi\,\hat{\mathbf{r}}+\cos\theta\cos\phi\,\hat{\boldsymbol{\theta}}-\sin\phi\,\hat{\boldsymbol{\phi}}\\\hat{\mathbf{y}}=\sin\theta\sin\phi\,\hat{\mathbf{r}}+\cos\theta\sin\phi\,\hat{\boldsymbol{\theta}}+\cos\phi\,\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}=\cos\theta\,\hat{\mathbf{r}}-\sin\theta\,\hat{\boldsymbol{\theta}}\end{cases}$$

$$\begin{cases}r=\sqrt{x^2+y^2+z^2}\\\theta=\tan^{-1}\left(\sqrt{x^2+y^2}/z\right)\\\phi=\tan^{-1}(y/x)\end{cases}\qquad\begin{cases}\hat{\mathbf{r}}=\sin\theta\cos\phi\,\hat{\mathbf{x}}+\sin\theta\sin\phi\,\hat{\mathbf{y}}+\cos\theta\,\hat{\mathbf{z}}\\\hat{\boldsymbol{\theta}}=\cos\theta\cos\phi\,\hat{\mathbf{x}}+\cos\theta\sin\phi\,\hat{\mathbf{y}}-\sin\theta\,\hat{\mathbf{z}}\\\hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}\end{cases}$$

Cylindrical

$$\begin{cases}x=s\cos\phi\\y=s\sin\phi\\z=z\end{cases}\qquad\begin{cases}\hat{\mathbf{x}}=\cos\phi\,\hat{\mathbf{s}}-\sin\phi\,\hat{\boldsymbol{\phi}}\\\hat{\mathbf{y}}=\sin\phi\,\hat{\mathbf{s}}+\cos\phi\,\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}=\hat{\mathbf{z}}\end{cases}$$

$$\begin{cases}s=\sqrt{x^2+y^2}\\\phi=\tan^{-1}(y/x)\\z=z\end{cases}\qquad\begin{cases}\hat{\mathbf{s}}=\cos\phi\,\hat{\mathbf{x}}+\sin\phi\,\hat{\mathbf{y}}\\\hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}\\\hat{\mathbf{z}}=\hat{\mathbf{z}}\end{cases}$$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l}=dx\,\hat{\mathbf{x}}+dy\,\hat{\mathbf{y}}+dz\,\hat{\mathbf{z}};\quad d\tau=dx\,dy\,dz$

Gradient: $$\nabla t=\frac{\partial t}{\partial x}\hat{\mathbf{x}}+\frac{\partial t}{\partial y}\hat{\mathbf{y}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla\cdot\mathbf{v}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}$$

Curl: $$\nabla\times\mathbf{v}=\left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}}+\left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}}+\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2t=\frac{\partial^2t}{\partial x^2}+\frac{\partial^2t}{\partial y^2}+\frac{\partial^2t}{\partial z^2}$$

Spherical. $d\mathbf{l}=dr\,\hat{\mathbf{r}}+r\,d\theta\,\hat{\boldsymbol{\theta}}+r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}};\quad d\tau=r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t=\frac{\partial t}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\boldsymbol{\theta}}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla\cdot\mathbf{v}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,v_\theta)+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl: $$\nabla\times\mathbf{v}=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,v_\phi)-\frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}}+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{\partial}{\partial r}(r\,v_\phi)\right]\hat{\boldsymbol{\theta}}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r\,v_\theta)-\frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2t=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2t}{\partial\phi^2}$$

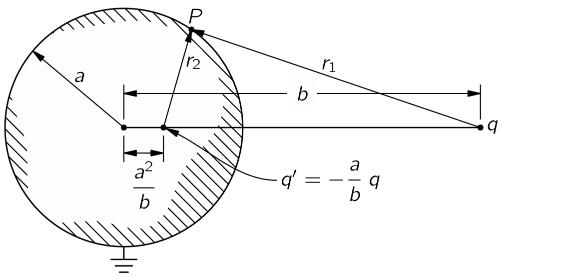
Cylindrical. $d\mathbf{l}=ds\,\hat{\mathbf{s}}+s\,d\phi\,\hat{\boldsymbol{\phi}}+dz\,\hat{\mathbf{z}};\quad d\tau=s\,ds\,d\phi\,dz$

Gradient: $$\nabla t=\frac{\partial t}{\partial s}\hat{\mathbf{s}}+\frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla\cdot\mathbf{v}=\frac{1}{s}\frac{\partial}{\partial s}(s v_s)+\frac{1}{s}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$

Curl: $$\nabla\times\mathbf{v}=\left[\frac{1}{s}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}}+\left[\frac{\partial v_s}{\partial z}-\frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}}+\frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi)-\frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2t=\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right)+\frac{1}{s^2}\frac{\partial^2t}{\partial\phi^2}+\frac{\partial^2t}{\partial z^2}$$



$$\boldsymbol{E} = \frac{q\boldsymbol{r}}{4\pi\varepsilon_0 r^3} = \frac{q\hat{\boldsymbol{r}}}{4\pi\varepsilon_0 r^2}$$

$$\boldsymbol{E} = \iiint_V \frac{\rho(\boldsymbol{r'})\mathrm{d}v}{4\pi\varepsilon_0} \cdot \frac{\boldsymbol{r} - \boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|^3}$$

$$\varphi = \frac{q}{4\pi\varepsilon_0 r}$$

$$\varphi = \iiint_V \frac{\rho(\boldsymbol{r'})\mathrm{d}v}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r'}|}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\oiint_S \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = \frac{Q}{\varepsilon_0}$$

$$\nabla \times \boldsymbol{E} = 0$$

$$\nabla \varphi = -\boldsymbol{E}$$

$$\nabla^2 \varphi = -\frac{\rho}{\varepsilon_0}$$

$$C = \frac{Q}{V}$$

$$\boldsymbol{p} = q\boldsymbol{d}$$

$$\varphi = \frac{\boldsymbol{p} \cdot \boldsymbol{r}}{4\pi\varepsilon_0 r^3}$$

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\boldsymbol{p} \cdot \boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\boldsymbol{p}}{r^3}\right)$$

$$E_\parallel = \frac{p}{4\pi\varepsilon_0} \cdot \frac{3\cos^2\theta - 1}{r^3}$$

$$E_\perp = \frac{p}{4\pi\varepsilon_0} \cdot \frac{3\sin\theta\cos\theta}{r^3}$$

$$E = \frac{p}{4\pi\varepsilon_0} \cdot \frac{\sqrt{3\cos^2 + 1}}{r^3}$$

$$d_{xy} = 6ql_1 l_2$$

$$\varphi_{xy} = \frac{3ql_1 l_2 xy}{4\pi\varepsilon_0 r^5}$$

$$d_{xx} = 6ql_1 l_2 = 6q(b^2 - a^2)$$

$$\varphi_{xx} = \frac{q(b^2 - a^2)(3x^2 - r^2)}{4\pi\varepsilon_0 r^5}$$

$$\varphi^{(n)} \propto r^{-n-1}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left(Q \cdot \frac{1}{r} - \boldsymbol{p} \cdot \nabla\frac{1}{r} + \frac{1}{6}\boldsymbol{D} : \nabla\nabla\frac{1}{r} - \cdots\right)$$

$$Q = \iiint_V \rho(\boldsymbol{r})\mathrm{d}v$$

$$\boldsymbol{p} = \iiint_V \boldsymbol{r}\rho(\boldsymbol{r})\mathrm{d}v$$

$$\boldsymbol{D}_{ij} = \iiint_V (3\boldsymbol{r}_i\boldsymbol{r}_j - \delta_{ij}r^2)\rho(\boldsymbol{r})\mathrm{d}v$$

$$\nabla\nabla\frac{1}{r} = \frac{1}{r^5}\begin{bmatrix} 3x^2 - r^2 & 3xy & 3xz \\ 3xy & 3y^2 - r^2 & 3yz \\ 3xz & 3yz & 3z^2 - r^2 \end{bmatrix}$$

$$\boldsymbol{E} = \frac{\sigma}{\varepsilon_0}\boldsymbol{n}$$

$$\boldsymbol{E}_{\text{surface}} = \frac{\sigma}{2\varepsilon_0}\boldsymbol{n}$$

$$\omega^2 = \frac{n_0 q_e^2}{\varepsilon_0 m_e}$$

$$U = \frac{1}{2}\iiint_V \varphi\rho\mathrm{d}v = \frac{\varepsilon_0}{2}\iiint_V E^2\mathrm{d}v$$

$$U = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

$$\boldsymbol{P} = Nq\boldsymbol{\delta}$$

$$\boldsymbol{P} = \chi\varepsilon_0\boldsymbol{E}$$

$$\sigma_{\text{pol}} = \boldsymbol{P} \cdot \boldsymbol{n} = P\cos\theta$$

$$\rho_{\text{pol}} = -\nabla \cdot \boldsymbol{P}$$

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$$

$$\iiint_S \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{s} = Q_{\text{free}}$$

$$\nabla \cdot \boldsymbol{D} = \kappa\varepsilon_0\nabla \cdot \boldsymbol{E} = \rho_{\text{free}}$$

$$\boldsymbol{P} = \left(1 - \frac{1}{\kappa}\right)\boldsymbol{D} = (\kappa - 1)\varepsilon_0\boldsymbol{E}$$

$$\boldsymbol{E} = \frac{q\boldsymbol{r}}{4\pi\kappa\varepsilon_0 r^3}$$

$$U = \frac{1}{2}\iiint_V \varphi\rho_{\text{free}}\mathrm{d}v = \frac{1}{2}\iiint_V \boldsymbol{D} \cdot \boldsymbol{E}\mathrm{d}v$$

$$\text{ball} : \begin{cases} E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 R^3} = \dfrac{\rho r}{3\varepsilon_0}, & r \le R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{\rho R^3}{3\varepsilon_0 r^2}, & r > R \end{cases} \\ \varphi = \begin{cases} \dfrac{Q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3} = \dfrac{\rho(3R^2 - r^2)}{6\varepsilon_0}, & r \le R \\ \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{\rho R^3}{3\varepsilon_0 r}, & r > R \end{cases} \\ U = \dfrac{3Q^2}{20\pi\varepsilon_0 R} = \dfrac{4\pi\rho^2 R^5}{15\varepsilon_0} \end{cases}$$

$$\text{sphere} : \begin{cases} E = \begin{cases} 0, & r < R \\ \dfrac{Q}{8\pi\varepsilon_0 r^2} = \dfrac{\sigma R^2}{2\varepsilon_0 r^2}, & r = R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{\sigma R^2}{\varepsilon_0 r^2}, & r > R \end{cases} \\ \varphi = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 R} = \dfrac{\sigma R}{\varepsilon_0}, & r \le R \\ \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{\sigma R^2}{\varepsilon_0 r}, & r > R \end{cases} \\ U = \dfrac{Q^2}{8\pi\varepsilon_0 R} = \dfrac{2\pi\sigma^2 R^3}{\varepsilon_0} \end{cases}$$

$$\text{wire} : \begin{cases} E = \dfrac{\lambda}{2\pi\varepsilon_0 r} \\ \varphi = -\dfrac{\lambda\ln r}{2\pi} \end{cases}$$

$$\text{plate} : \begin{cases} E = \dfrac{\sigma}{2\varepsilon_0} \\ C = \dfrac{\varepsilon_0 S}{d} \\ U = \dfrac{Q^2 d}{2\varepsilon_0 S} \\ \sigma_U = \dfrac{\sigma^2 d}{2\varepsilon_0} \end{cases}$$

$$\text{cylinder} : \begin{cases} C = \dfrac{2\pi\varepsilon_0 l}{\ln(r_2/r_1)} \\ U = \dfrac{Q^2}{4\pi\varepsilon_0 l}\ln\dfrac{r_2}{r_1} \\ \lambda_U = \dfrac{\lambda^2}{4\pi\varepsilon_0}\ln\dfrac{r_2}{r_1} \end{cases}$$

$$\sigma_0\cos\theta : \begin{cases} \dfrac{\sigma_0}{3\varepsilon_0}\boldsymbol{e}_x, & r < R \\ \dfrac{R^3\sigma_0}{3\varepsilon_0}\dfrac{3\boldsymbol{r}_x\boldsymbol{r} - r^2\boldsymbol{e}_x}{r^5} = \dfrac{R^3\sigma_0}{\varepsilon_0 r^3}\left(\cos\theta - \dfrac{1}{3}, \cos\theta\sin\theta\right), & r > R \end{cases}$$